Imprinting primordial gravitational waves on the map of cosmic microwave background

蔡一夫 Yi-Fu Cai

Lecture for the Frontiers of Astrophysics @ Peking U

March 13th 2018

中国科学技术大学天文学系

Department of Astronomy, University of Science and Technology of China

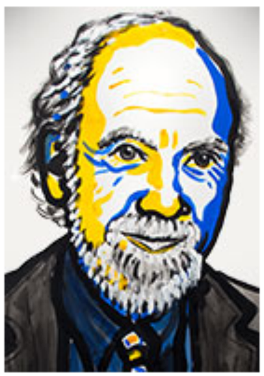
Shocking (but old) news !

The Nobel Prize in Physics 2017
Rainer Weiss, Barry C. Barish, Kip S. Thorne

Share this:

The Nobel Prize in Physics 2017

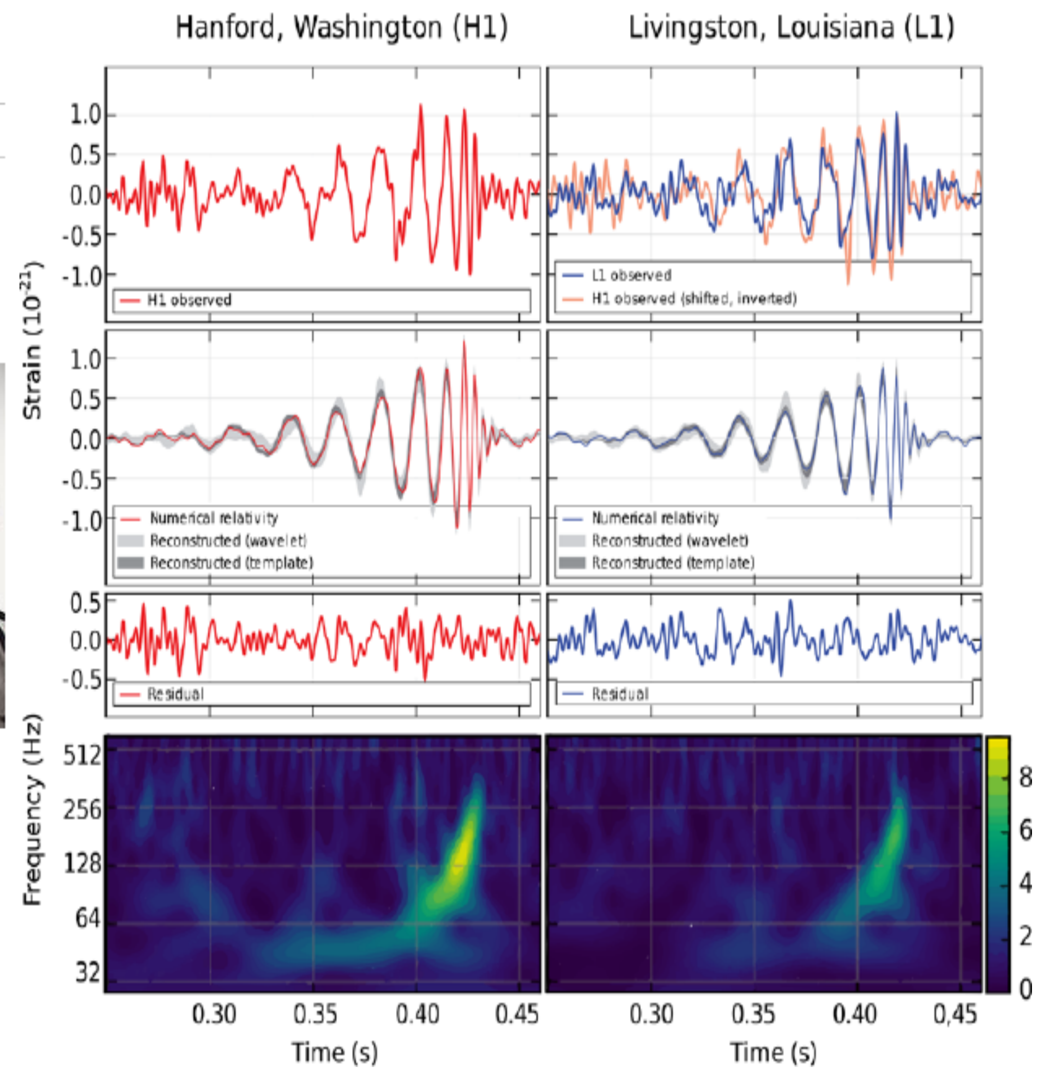
© Nobel Media. Ill. N. Elmehed
Rainer Weiss
Prize share: 1/2



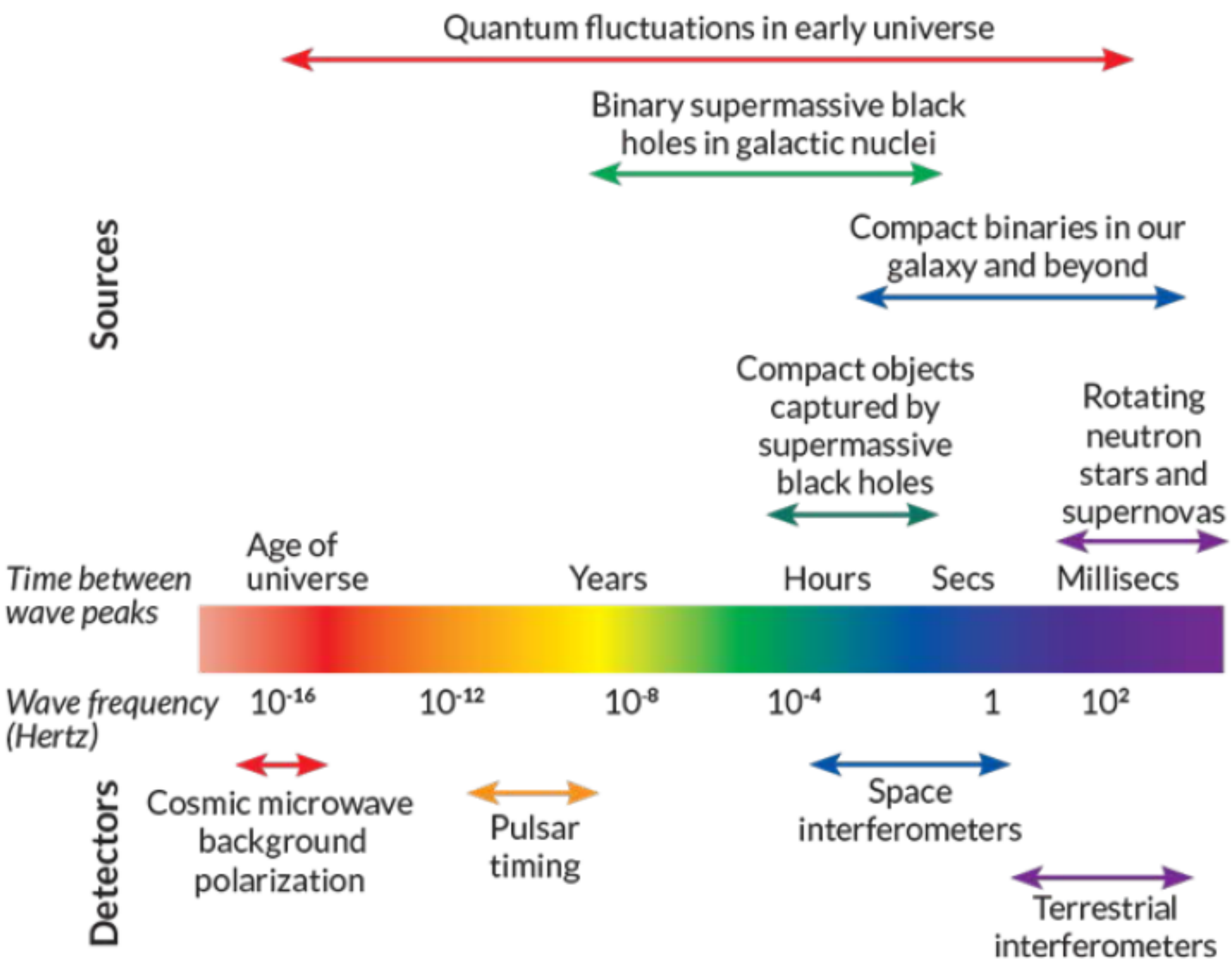
© Nobel Media. Ill. N. Elmehed
Barry C. Barish
Prize share: 1/4

© Nobel Media. Ill. N. Elmehed
Kip S. Thorne
Prize share: 1/4

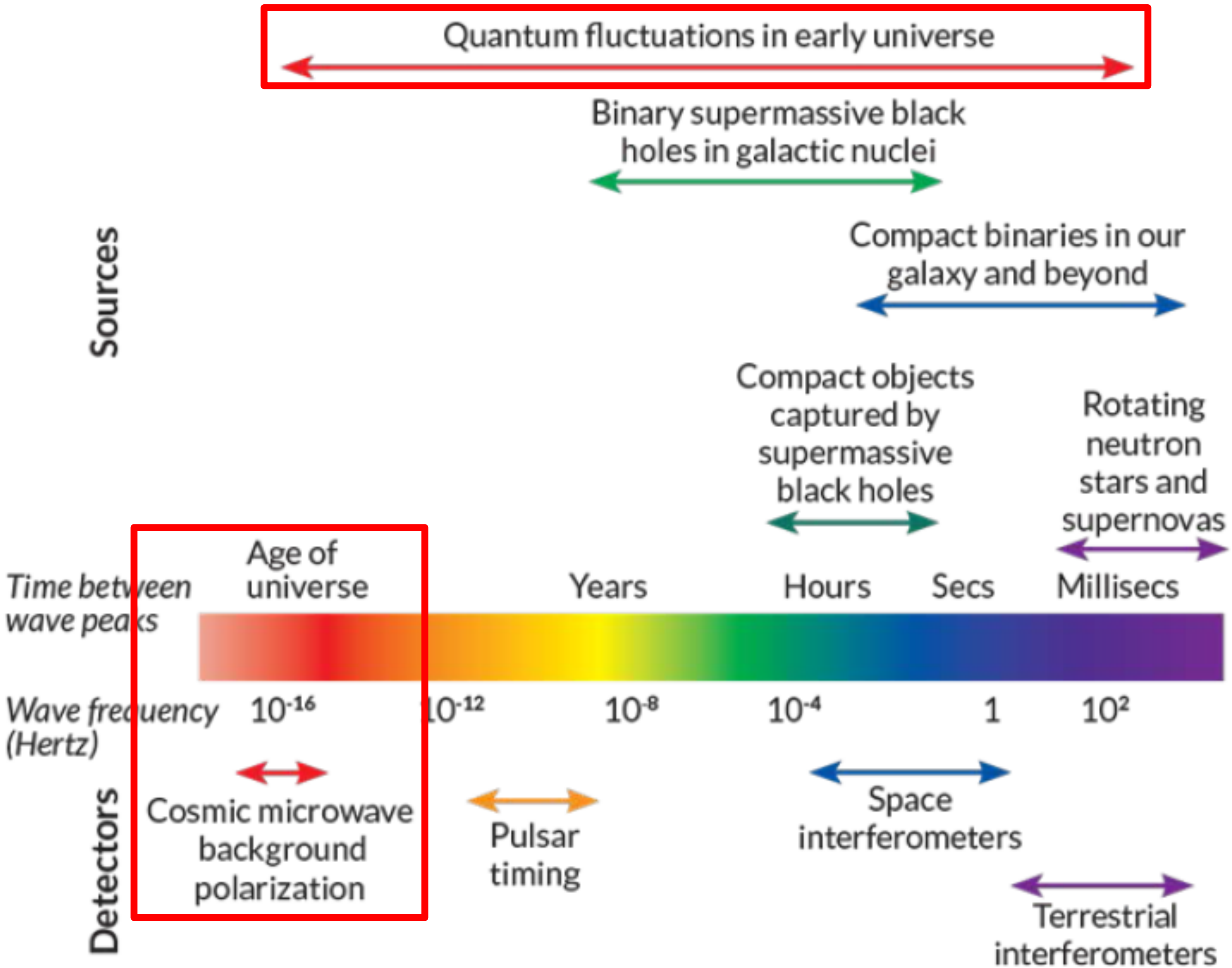
The Nobel Prize in Physics 2017 was divided, one half awarded to Rainer Weiss, the other half jointly to Barry C. Barish and Kip S. Thorne *"for decisive contributions to the LIGO detector and the observation of gravitational waves"*.



Gravitational Waves in the Universe



Today's story: Primordial GWs



Motivation

Primordial GWs can be a powerful probe of the very early universe:

- Produced in very early times (within 10^{-30}s after big bang)
  - Inflationary cosmology
  - Bouncing cosmology
  - …
- Linearly decoupled from other matter fields
  - Objective information recorder
  - Purity of the power spectrum
- Originated from quantum fluctuations of spacetime
  - Indicator to quantum gravity

Outline

✧ Part I: Theory of PGWs

- Classical Tensor Perturbations

- Quantization and Power Spectrum

- Perturbation Theories for
    - Inflationary cosmology
    - Bouncing cosmology

Outline

- ✧ Part II: From PGWs to the CMB
  - Introduction to CMB
  - Harmonic Analysis for CMB Polarizations
  - E and B modes from PGWs
  - Lensing induced and foreground B modes
- ✧ Summary

Part I: Theory of PGWs

- Classical Tensor Perturbations
- Quantization and Power Spectrum
- Perturbation Theories for
  - Inflationary cosmology
  - Bouncing cosmology

Classical Tensor Perturbations

- Consider a spatially flat Friedmann-Robertson-Walker background:
  - In cosmic time
$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$
  - In conformal time
$$ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j)$$
- Tensor perturbations are **traceless** & **transverse** :
$$\delta_{ij} \to \delta_{ij} + h_{ij} \qquad h_{ij} = h_{ji} \ ; \ h_{ii} = 0 \ ; \ h_{ij,j} = 0$$
- Linearized Einstein equations (synchronous gauge)

Anisotropic stress

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi G a^2 \boxed{(T^i_j - P\delta_{ij})}$$

- Classical evolution is similar to the massless scalar field case, despite of the spacetime index.

Classical Tensor Perturbations

- Fourier expansion:

$$h_{ij}(x) = \sqrt{16\pi G} \sum_r \int \frac{d^3k}{(2\pi)^3} \epsilon^r_{ij}(\mathbf{k}) h^r_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}$$

where the symmetric polarization tensor is traceless & transverse, and is normalized as

$$\sum_{ij} \epsilon^r_{ij}(\mathbf{k}) \epsilon^s_{ij}(\mathbf{k})^* = 2\delta^{rs}$$

- Equation of motion in Fourier space (no source term):

$$h^{r\,\prime\prime}_{\mathbf{k}} + 2\frac{a'}{a} h^{r\,\prime}_{\mathbf{k}} + k^2 h^r_{\mathbf{k}} = 0$$

- Assuming a power law expansion:

$$a \propto \tau^n \quad n = \frac{2}{1+3w} \quad w = P/\rho$$

namely, w=1/3 for radiation; w=0 for dust matter; w~-1 for inflation

Classical Tensor Perturbations

- Equation of motion with a constant background equation of state

$$h_{\mathbf{k}}^{r\,\prime\prime} + 2\frac{n}{\tau}h_{\mathbf{k}}^{r\,\prime} + k^2 h_{\mathbf{k}}^r = 0$$

- General solutions in terms of Bessel functions:

$$h_k(\tau) = \tau^{1-n}\left(Aj_{\nu-1/2}(k\tau) + By_{\nu-1/2}(k\tau)\right)$$

$$\nu^2 = n(n-1) + \frac{1}{4}$$

- A little mathematical properties:

For large z:

$$J_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\cos\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\operatorname{Im}(z)|}O(|z|^{-1})\right)$$

$$Y_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\sin\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\operatorname{Im}(z)|}O(|z|^{-1})\right)$$

For small & negative z:

$$J_\alpha(z) \sim \frac{(-1)^\alpha}{(-\alpha)!}\left(\frac{2}{z}\right)^\alpha$$

$$Y_\alpha(z) \sim -\frac{(-1)^\alpha\Gamma(-\alpha)}{\pi}\left(\frac{z}{2}\right)^\alpha$$

- Thus, general solutions have two asymptotical forms:
  - When |k\tau| >> 1, oscillatory behaviors
  - When |k\tau| << 1, constant and time-varying modes

Part I: Theory of Primordial GWs

- Classical Tensor Perturbations
- Quantization and Power Spectrum
- Perturbation Theories for
    - Inflationary cosmology
    - Bouncing cosmology

Quantization and Power Spectrum

- In the linear theory, one can take the analogy with the massless scalar field case to build the quantum theory associated with the tensor modes in a curved spacetime.

$$h_{ij}(\mathbf{x},\tau) = \sum_r \sqrt{16\pi G}\int \frac{d^3k}{(2\pi)^3}\left[\epsilon^r_{ij}(\mathbf{k})\underbrace{h_k(\tau)a^r_{\mathbf{k}}}_{\text{physical time-dependent operator}}e^{i\mathbf{k}\cdot\mathbf{x}} + \epsilon^r_{ij}(\mathbf{k})^* h_k(\tau)^* a^{r\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

- The conjugate momenta are expressed as

$$\pi_{ij} = \frac{\delta^{(2)}S}{\delta h'_{ij}} = \frac{1}{2}a^2 h'_{ij}$$

- Quantized Poisson brackets

$$\begin{aligned}
[h_{ij}(\tau,\mathbf{k}), h_{kl}(\tau,\mathbf{k}')] &= 0\ , \\
[p_{ij}(\tau,\mathbf{k}), p_{kl}(\tau,\mathbf{k}')] &= 0\ , \\
[h_{ij}(\tau,\mathbf{k}), p_{kl}(\tau,\mathbf{k}')] &= \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \\
&\times\ \delta^{(3)}(\mathbf{k}-\mathbf{k}')\ .
\end{aligned}$$

Wronskian normalization condition

$$\boxed{h_k\dot{h}^*_k - h^*_k\dot{h}_k = \frac{i}{a^2}}$$

Quantization and Power Spectrum

- Power spectrum is defined by

$$\langle 0|h_{ij}(\mathbf{x},\tau)h_{ij}(\mathbf{y},\tau)|0\rangle \equiv \int \frac{d^3k}{(2\pi)^3}\frac{2\pi^2}{k^3}\Delta_T^2(k,\tau)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

- Or, it is simply expressed as

$$\boxed{\Delta_T^2(k,\tau) = 64\pi G\frac{k^3}{2\pi^2}|h_k(\tau)|^2}$$

- Energy spectrum is associated with the energy density via:

$$T_{GW}^{\mu\nu} = -\frac{2}{\sqrt{\bar{g}}}\frac{\delta S_{GW}}{\delta \bar{g}_{\mu\nu}} \qquad \rho_{GW} = T_{GW}{}^0{}_0 = \bar{g}_{00}T_{GW}^{00}$$

$$\Omega_{GW}(k,\tau) \equiv \frac{1}{\rho_c(\tau)}\frac{d\langle 0|\rho_{GW}|0\rangle}{d(\ln k)} = \frac{8\pi G}{3H(\tau)^2}\frac{k^3}{2\pi^2 a^2(\tau)}\left(|\dot{h}_k(\tau)|^2 + k^2|h_k|^2\right)$$

Quantization and Power Spectrum

- Comments:
  - Primordial GWs could be originated from quantum fluctuations of the spacetime of the baby universe such that they can be accommodated with the quantum theory;
  - These quantum mechanically generated spacetime ripples need to be squeezed into classical perturbations so that the power and energy spectra can be probed observationally;
  - The above two theoretical expectations happen to be in agreement with two asymptotical solutions for primordial GWs in forms of the Bessel functions
- Question: Is there any causal mechanism that can connect two asymptotical solutions of primordial GWs dynamically?

Part I: Theory of Primordial GWs

- Classical Tensor Perturbations
- Quantization and Power Spectrum
- Perturbation Theories for
  - Inflationary cosmology
  - Bouncing cosmology

Inflationary Cosmology

Consider a Lagrangian of a free massive scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

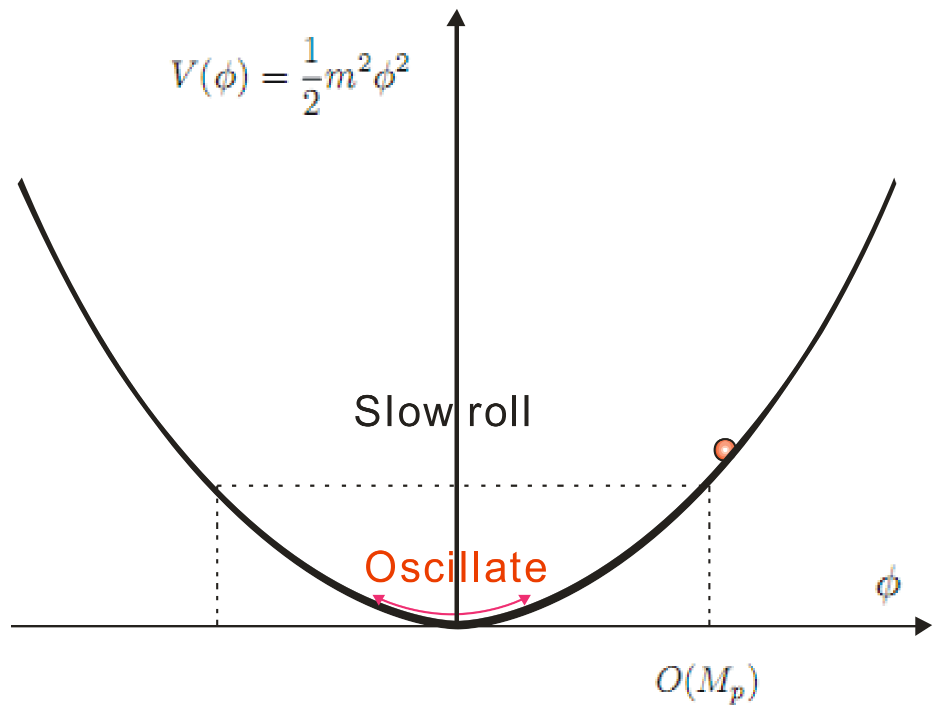
Its cosmological evolution follows the Friedmann equation and the Klein-Gordon equation:

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

This model yields an accelerating phase at high energy scale, when the amplitude of the scalar is larger than the Planck mass.

Inflation



Guth '81; Linde '82;
Albrecht & Steinhardt, '82;
Starobinsky '80; Sato '81; Fang '81; …

Inflationary Cosmology

- The horizon, flatness, monopole problems can be solved
- Primordial fluctuations can lead to the formation of LSS
- Nearly scale-invariant power spectrum of primordial density fluctuations
- Small amount of primordial non-Gaussianities
- Sizable tensor modes
- Trans-Planckian perturbations?
- Initial big bang singularity?

PGWs of Inflationary Cosmology

- Slow roll inflation yields a phase of nearly exponential expansion (\tau < 0):

$$a \propto e^{Ht} \sim -\frac{1}{H\tau} \quad \Rightarrow \quad \frac{a''}{a} = \frac{2}{\tau}$$

- Slow roll parameters are introduced by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta \equiv M_p^2 \frac{V_{,\phi\phi}}{V}$$

and are much less than unity.

- Mukhanov-Sasaki variable and the EoM:

$$v_k \equiv h_{\mathbf{k}}/a \qquad v_k'' + (k^2 - \frac{a''}{a})v_k = 0$$

- Applying the vacuum initial condition, the solution for PGWs during inflation is given by

$$\boxed{h_k(\tau) = -\frac{H}{\sqrt{2k}}\tau\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau} = H\sqrt{\frac{k}{2}}\tau^2 h_1^{(2)}(k\tau)}$$

PGWs of Inflationary Cosmology

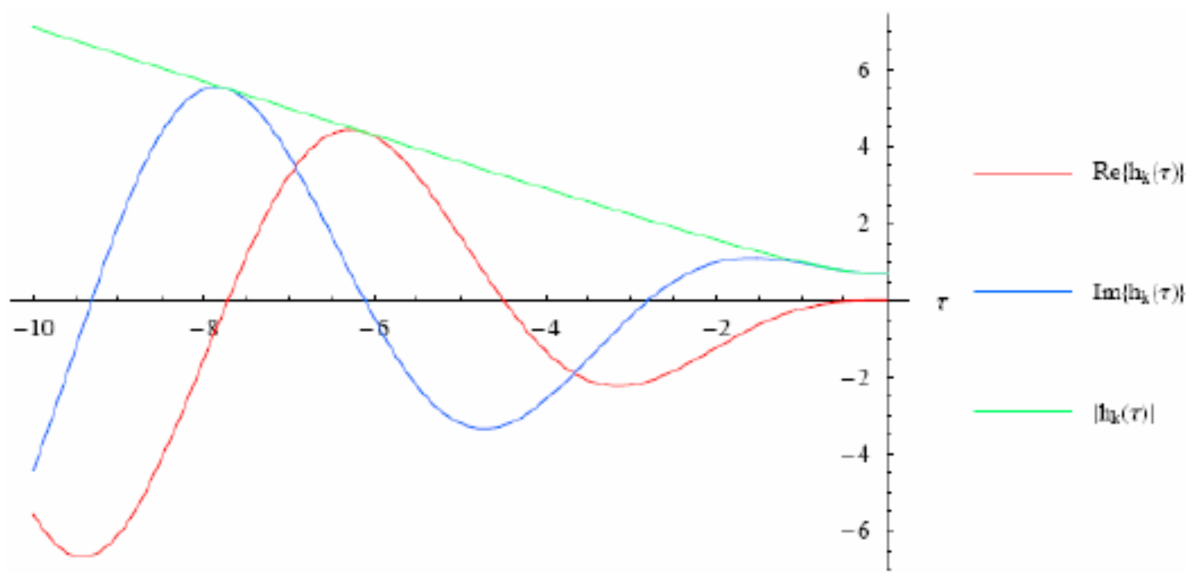
- When |k\tau|>>1, the sub-Hubble solution connects with the vacuum fluctuations

$$h_{\mathbf{k}}^{\mathrm{UV}} \to -\frac{H\tau}{\sqrt{2k}} e^{-ik\tau}$$

- When |k\tau|<<1, the super-Hubble solution get frozen to become constant mode

$$h_{\mathbf{k}}^{\mathrm{IR}} \to \frac{iH}{\sqrt{2k^3}}$$

- Hubble crossing condition: k = aH or |k\tau| = 1
- For a k=1 mode, the solution is numerically given in the following



PGWs of Inflationary Cosmology

- At the end of inflation (epsilon=1), for the tensor modes outside the Hubble radius:
  - Power spectrum: $$P_T \equiv \Delta_T^2(k, \tau_e) = \frac{2H_*^2}{\pi^2 M_p^2}$$
  - Spectral index: $$n_T \equiv \frac{d \ln P_T}{d \ln k} = -2\epsilon_*$$
  - Tensor-to-Scalar ratio: $$r \equiv \frac{P_T}{P_\zeta} = 16\epsilon_* \quad \Rightarrow \quad \boxed{r = -8n_T}$$

    Consistency relation for the model of single field slow roll inflation
  - Energy spectrum: $$\Omega_{GW}(k, \tau_e) = \frac{H^2}{6\pi^2 M_p^2} = \frac{1}{12}\Delta_T^2(k, \tau_e)$$
    $$\Omega_{GW}(k, \tau_{\rm today}) = T(k; \tau_{\rm today}, \tau_e) \times \Omega_{GW}(k, \tau_e)$$
- Today's energy spectrum can be related to the primordial energy spectrum via the transfer function.

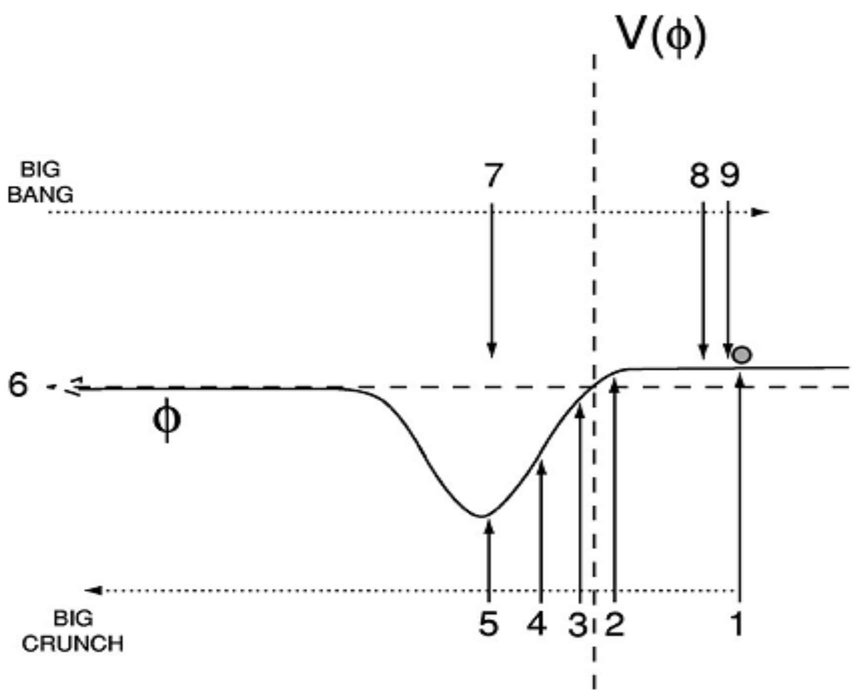
Part I: Theory of Primordial GWs

- Classical Tensor Perturbations
- Quantization and Power Spectrum
- Perturbation Theories for
  - Inflationary cosmology
  - Bouncing cosmology

Ekpyrotic Bounce

The collision of two M branes in 5D gives rise to a nonsingular cyclic universe, and the description of EFT in 4D is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \mathcal{R} + \frac{1}{2} (\partial\phi)^2 - V(\phi) + \beta^4(\phi)(\rho_M + \rho_R) \right)$$



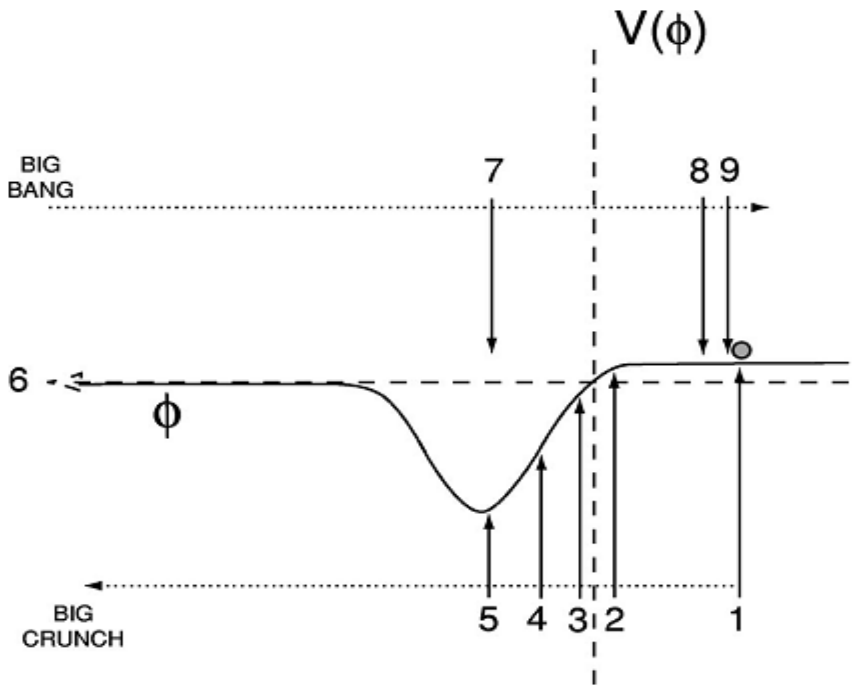
1 DE domination
2 decelerated expansion
3 turnaround
4 ekpyrotic contracting phase
5 before big crunch
6 a singular bounce in 4D
7 after big bang
8 radiation domination
9 matter domination

Khoury, Ovrut, Steinhardt & Turok, PRD, '01

Ekpyrotic Bounce

The collision of two M branes in 5D gives rise to a nonsingular cyclic universe, and the description of EFT in 4D is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \mathcal{R} + \frac{1}{2} (\partial\phi)^2 - V(\phi) + \beta^4(\phi)(\rho_M + \rho_R) \right)$$



1 DE domination
2 decelerated expansion
3 turnaround
4 ekpyrotic contracting phase
5 before big crunch
6 a **singular** bounce in 4D
7 after big bang
8 radiation domination
9 matter domination

Failure of effective field theory description, uncertainty involved in perturbations.

Ekpyrotic Bounce

Predictions by Ekpyrotic model:

- Nearly scale-invariant power spectrum
- Large primordial non-Gaussianities
- Almost no tensor modes

Nonsingular Bounce

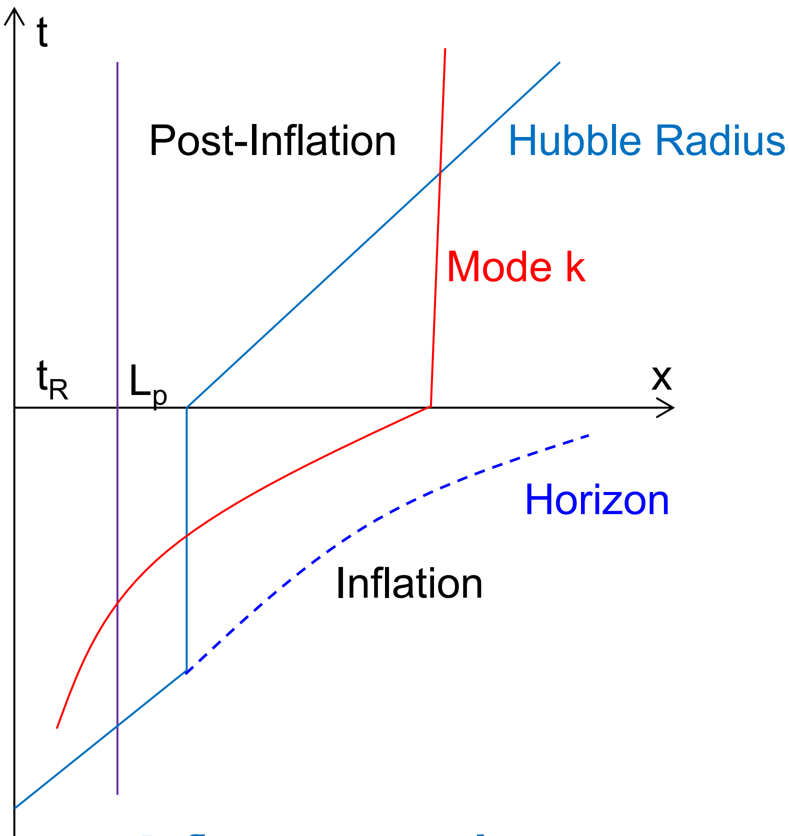
- Pre-Big-Bang
  (non-perturbative effects) Gasperini & Veneziano ’92
- String Gas Cosmology
  (thermal non-local system) Brandenberger & Vafa ’89
- *Matter Bounce* Cosmology
  (NEC violating matter. Wands ’98; Finelli & Brandenberger ’01)
- Loop Quantum Cosmology
  (quantum structure of spacetime. Ashtekar 0812.0177)
- Mirage Cosmology
  (nonconventional braneworld. Kehagias & Kiritsis ’99)
- Lee-Wick Bounce
  (high-order derivatives. *CYF*, Qiu, Brandenberger & Zhang ’08)
- New Ekpyrotic Cosmology
  (ghost condensate. Buchbinder, Khoury & Ovrut ’07; Creminelli, Luty, Nicolis & Senatore ’06)
- Galilean Genesis
  (galilean field. Creminelli, Nicolis & Trincherini ’10)
- …

Sketch Plot

Crucial facts:

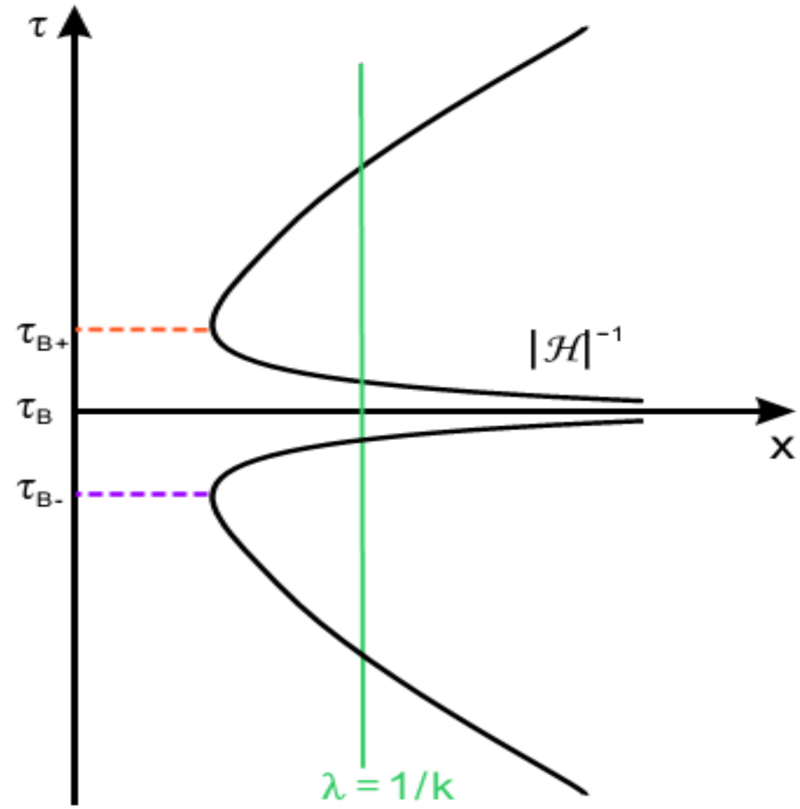
- Fluctuations originate on sub-Hubble scales
- Fluctuations propagate for a long time on super-Hubble scales
- Trans-Planckian problem: Inflation; Bounce

Brandenberger, hep-ph/9910410



Inflationary prediction:
a scale-invariant spectrum

CYF, Qiu, Brandenberger & Zhang, 0810.4677



Can we obtain a scale-invariant spectrum as in inflation?

PGWs of Matter Bounce Cosmology

- Matter bounce requires a phase of matter-dominated contraction (w=0, \tau < 0):

$$a \sim \tau^2 \quad \Rightarrow \quad \frac{a''}{a} = \frac{2}{\tau}$$

- Similar to the slow roll, there is

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}(1+w)$$

- Inserting to the EoM of Mukhanov-Sasaki variable

$$v_k \equiv h_{\mathbf{k}}/a \qquad v_k'' + (k^2 - \frac{a''}{a})v_k = 0$$

- Using the vacuum initial condition, the solution for PGWs in matter contraction takes

$$\begin{aligned} v = (\eta - \tilde{\eta}_{B-})^{1/2}\{A_k^T J_{-(3/2)}[k(\eta - \tilde{\eta}_{B-})] \\ + B_k^T J_{3/2}[k(\eta - \tilde{\eta}_{B-})]\}, \end{aligned}$$

$$A_k^T = i\frac{\sqrt{\pi}}{2} \quad \text{and} \quad B_k^T = -\frac{\sqrt{\pi}}{2}$$

PGWs of Matter Bounce Cosmology

- Therefore, the asymptotic form of the solution in matter contracting phase is

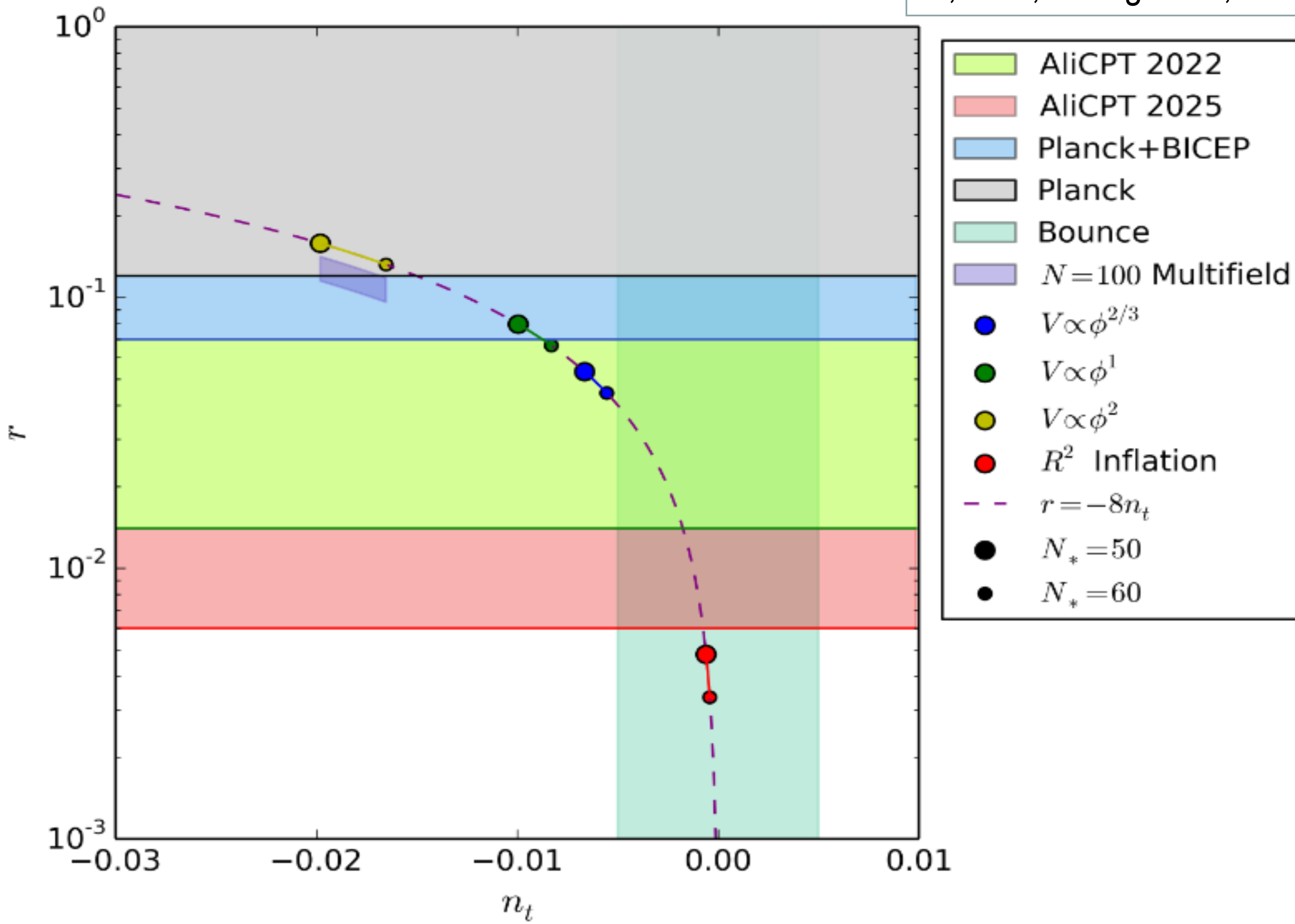
$$v(k,\eta) = \begin{cases} -\frac{i}{\sqrt{2}} k^{-(3/2)} (\eta - \tilde{\eta}_{B-})^{-1}, & \text{outside Hubble radius} \\ \frac{1}{\sqrt{2k}} e^{-ik(\eta - \tilde{\eta}_{B-})}, & \text{inside Hubble radius} \end{cases}$$

- After evolving PGWs through the bouncing phase to connect with thermal expansion,
  - Power spectrum: $$P_T(k) = G\frac{32k^3}{\pi}\left|\frac{v^f}{a}\right|^2 = \frac{2\rho_{B+}}{27\pi^2 M_p^4}$$
  - Spectral index: $$n_T \equiv \frac{d\ln P_T}{d\ln k} = 0 \pm (\text{bg error})$$
  - Tensor-to-Scalar ratio: $$r \equiv \frac{P_T}{P_\zeta} \lesssim O(1)$$

Consistency relation is broken for matter bounce

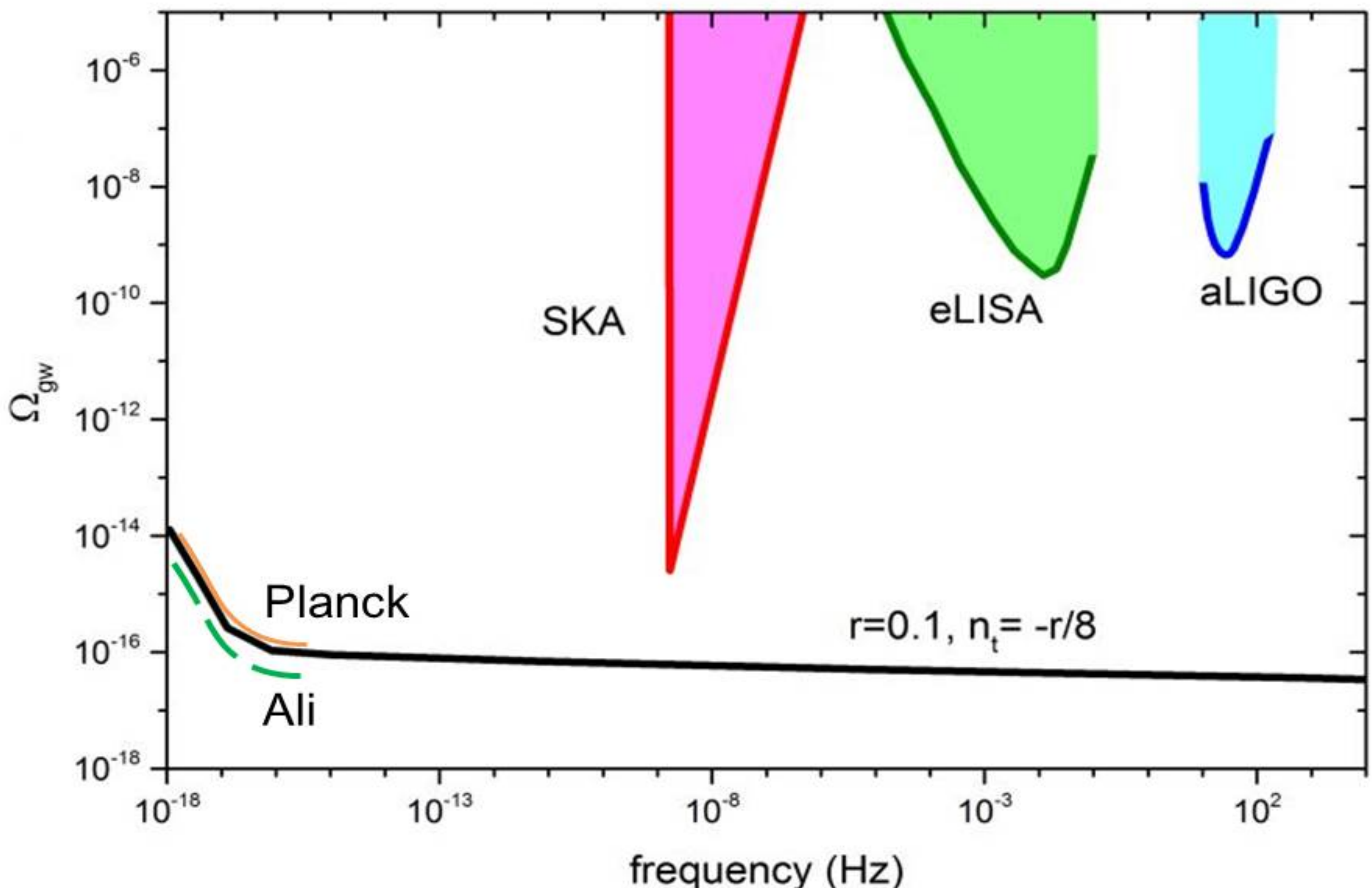
Consistency relation of inflationary PGWs and comparison with matter bounce and experiments

CYF & Zhang, 1605.01840;
Li, *CYF*, Zhang et.al., 1710.03047



Energy spectrum of inflationary PGWs and comparison with experimental sensitivities

CYF & Zhang, 1605.01840



Evolutions after primordial era

- Based on the big bang theory, assuming a CDM model:

  Radiation + Matter + instantaneous transition

$$a(\tau) = \begin{cases} H_0\sqrt{\Omega_{r0}}\tau, & 0 \le \tau \le \tau_{eq} \\ a_{eq}\left(\frac{\tau}{\tau_{eq}}\right)^2, & \tau_{eq} < \tau \le \tau_0 \end{cases}$$

- Solutions:

  - Radiation expansion

$$h_k(\tau) = Aj_0(k\tau) + By_0(k\tau) \longrightarrow A = h_k(0), \;\; B = 0$$

  - Matter expansion

$$h_k(\tau) = A_k\left(\frac{3j_1(k\tau)}{k\tau}\right) + B_k\left(\frac{3y_1(k\tau)}{k\tau}\right)$$

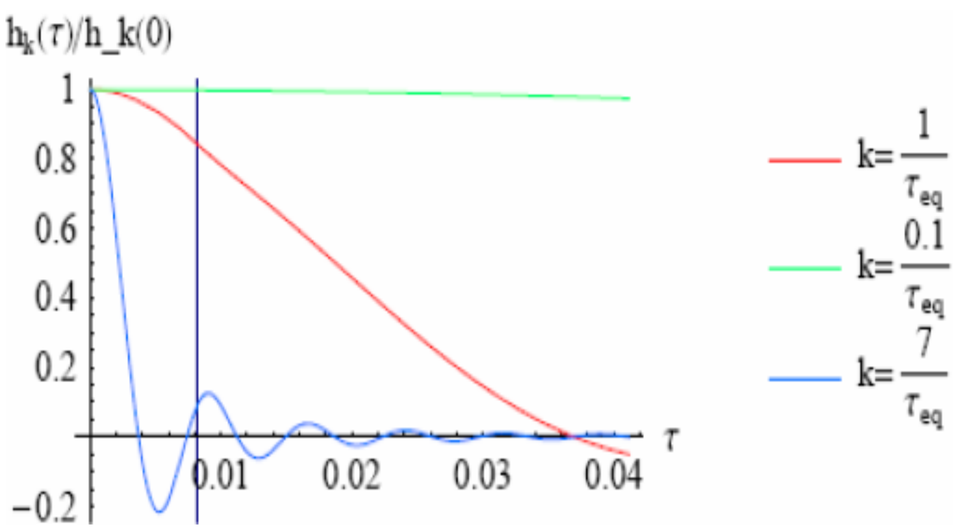
Evolutions after primordial era

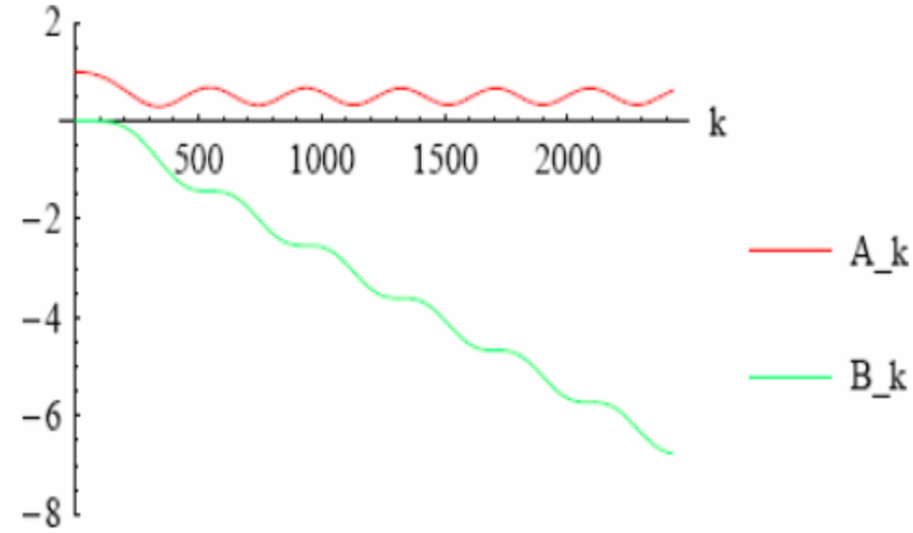
- Continuity of the solution yields the coefficients:

$$A_k = h_k(0)\frac{3k\tau_{eq} - k\tau_{eq}\cos(2k\tau_{eq}) + 2\sin(2k\tau_{eq})}{6k\tau_{eq}}$$

$$B_k = h_k(0)\frac{2 - 2k^2\tau_{eq}^2 - 2\cos(2k\tau_{eq}) - k\tau_{eq}\sin(2k\tau_{eq})}{6k\tau_{eq}}$$

- Modes reenter the Hubble radius during radiation and matter phases:





- The transfer coefficients would be more complicated if more elements are taken into account, namely, the dark energy era, smooth transitions, anisotropic stress (cosmic neutrinos), etc.

Evolutions after primordial era

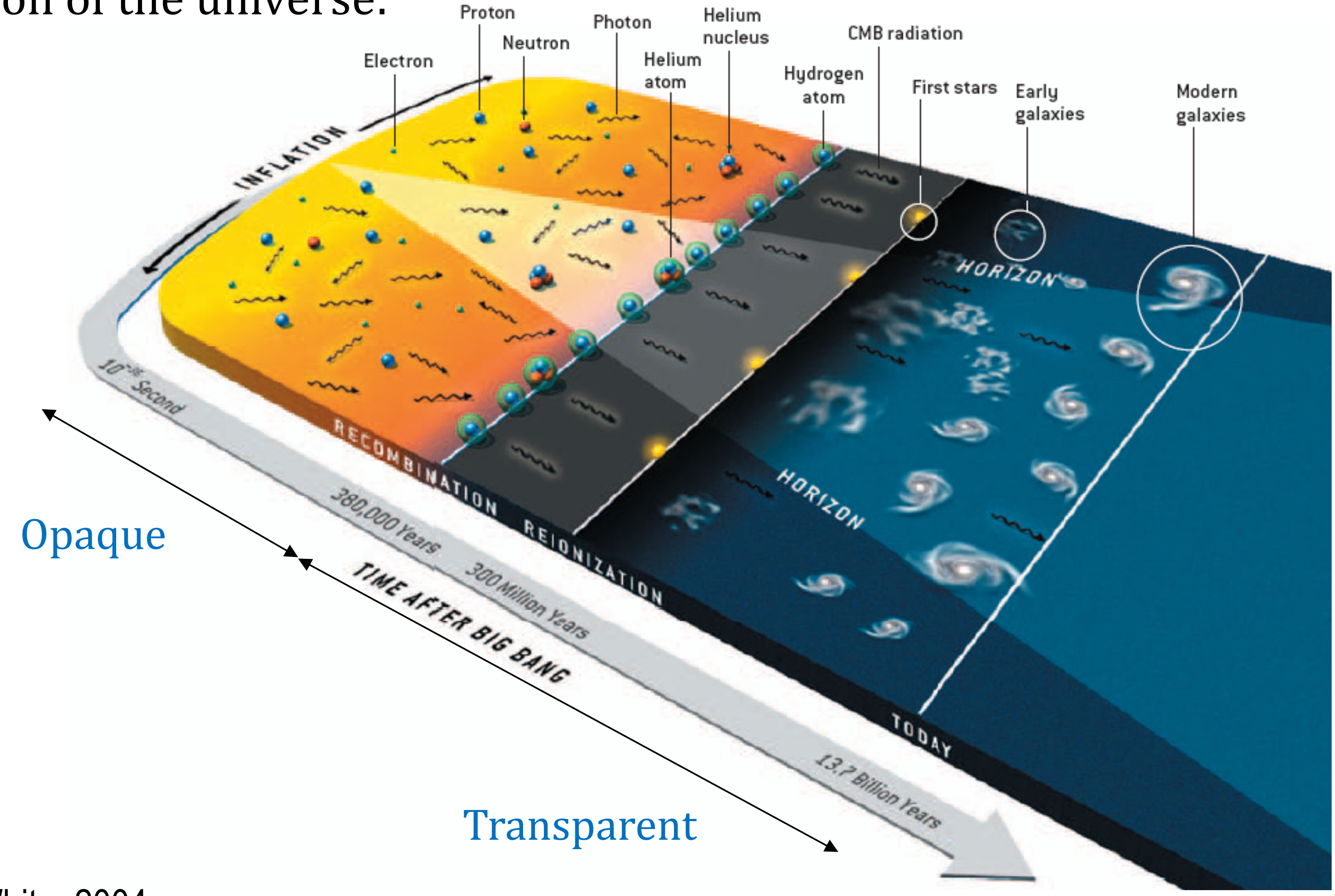
- Comments:
  - The generation of PGWs are associated with the environment of the primordial phase;
  - Evolutions after the primordial era, i.e. transfer functions, are associated with phases of radiation, matter, dark energy, and phase transitions that the universe has experienced, as well as the relativistic d.o.f., such as free-streaming neutrinos;
  - The study of the cosmic background of PGWs can reveal important information about the evolution of the universe throughout the whole history.
- Question: How can we probe them in cosmological observations?

Part II: From PGWs to the CMB

- Introduction to CMB
- Harmonic Analysis for CMB Polarizations
- E and B modes from PGWs
- Lensing induced and foreground B modes

Introduction to CMB

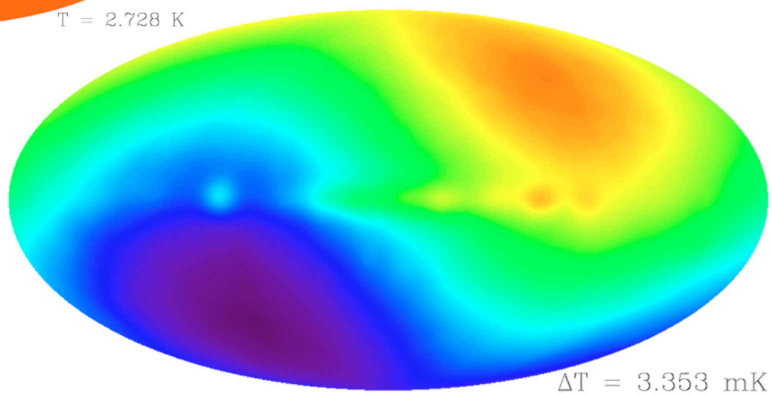
Evolution of the universe:



Hu & White, 2004

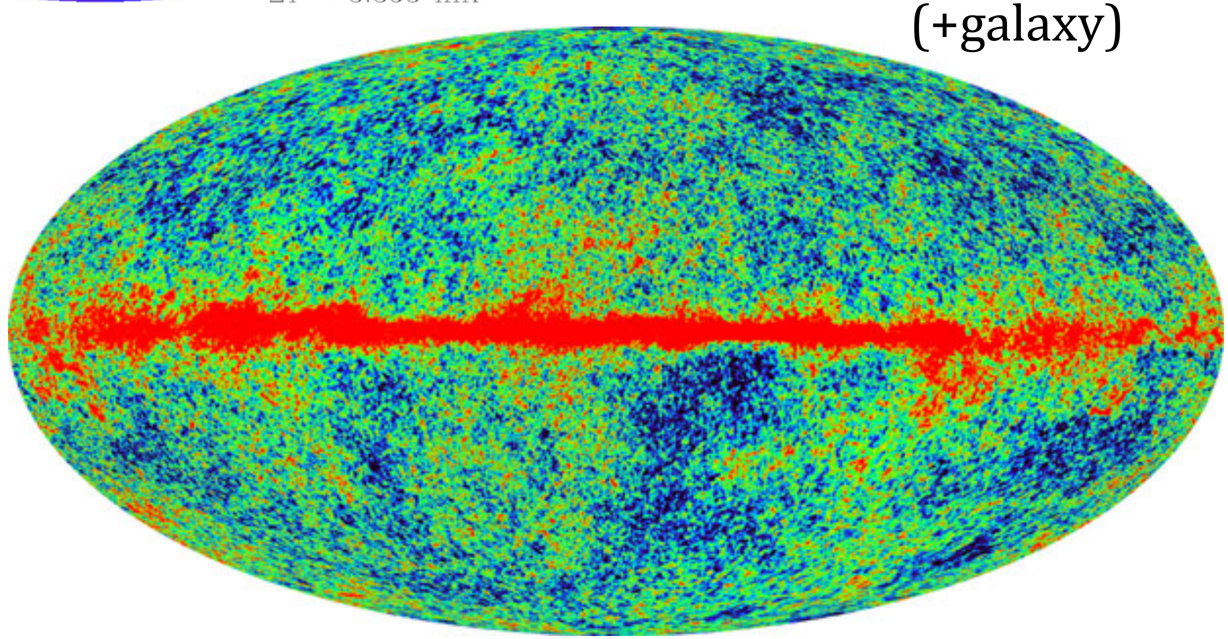
Introduction to CMB

(almost) uniform 2.725K blackbody spectrum



Dipole (local motion)

O(10^{-5}) perturbations
(+galaxy)

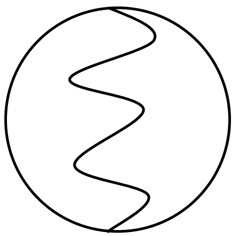


Observations:
Today's microwave sky

Credit: NASA/WMAP Team

Primordial origin of temperature fluctuations

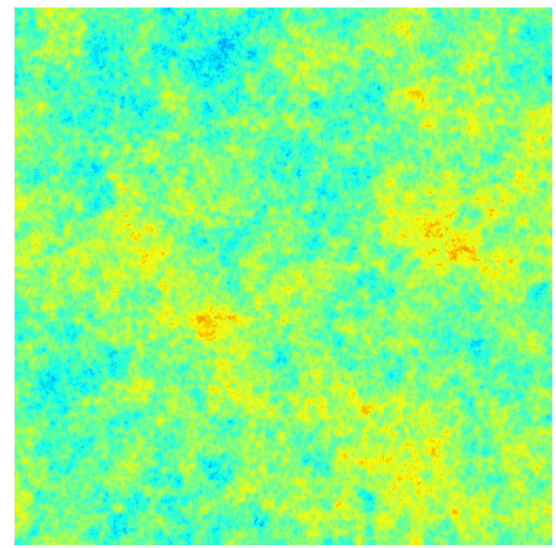
A first glance at perturbation theory in inflation



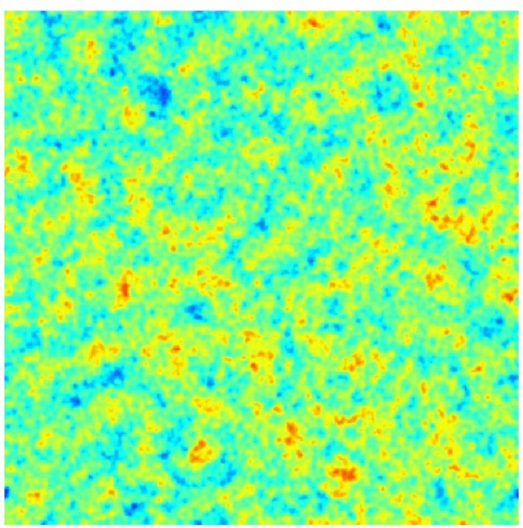
Quantum fluctuations
"waves in a pocket universe"

Inflation
$>10^{30}$ times bigger

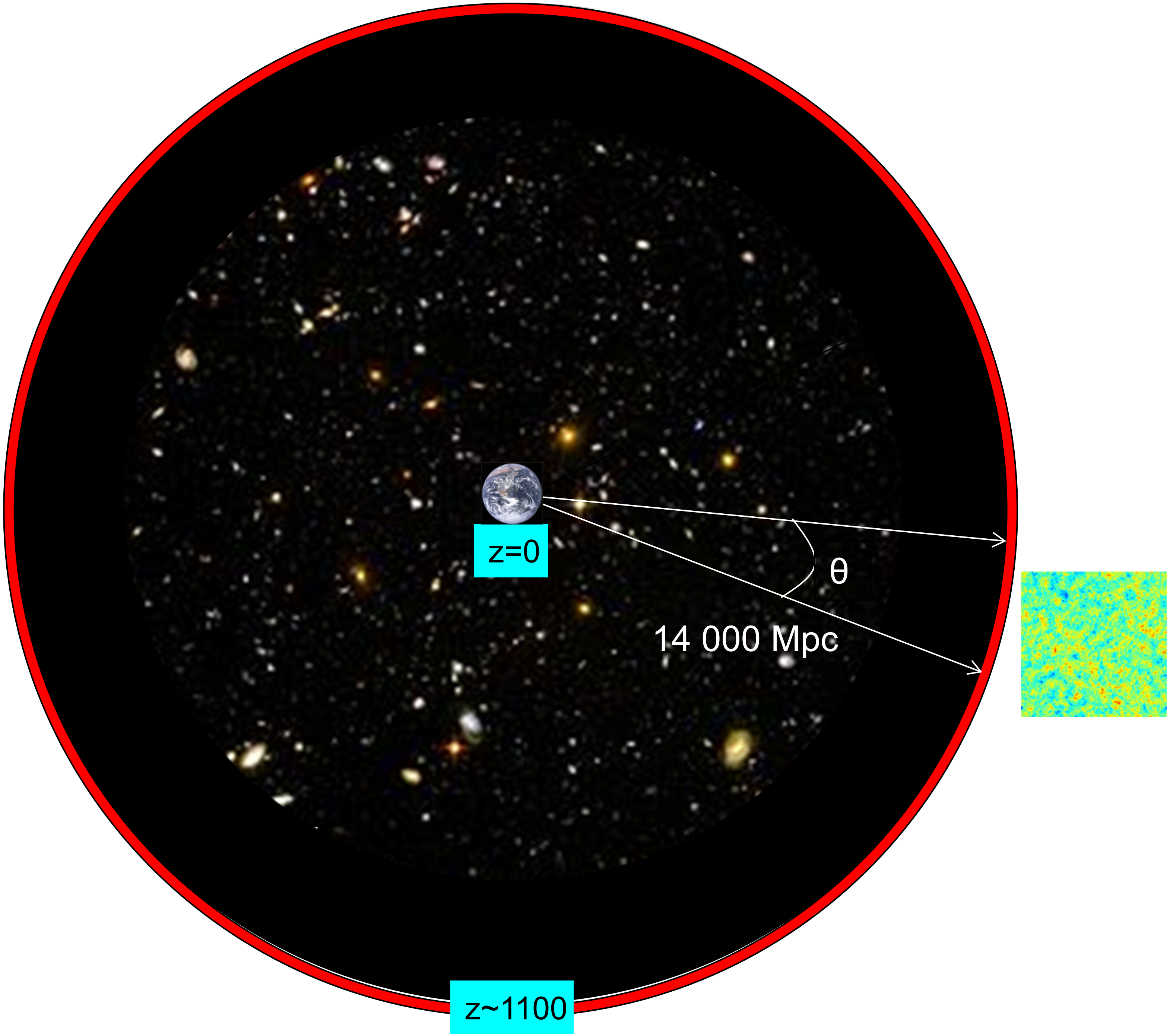
End of inflation
Huge size, amplitude $\sim 10^{-5}$



+ gravity
+ pressure
+ diffusion

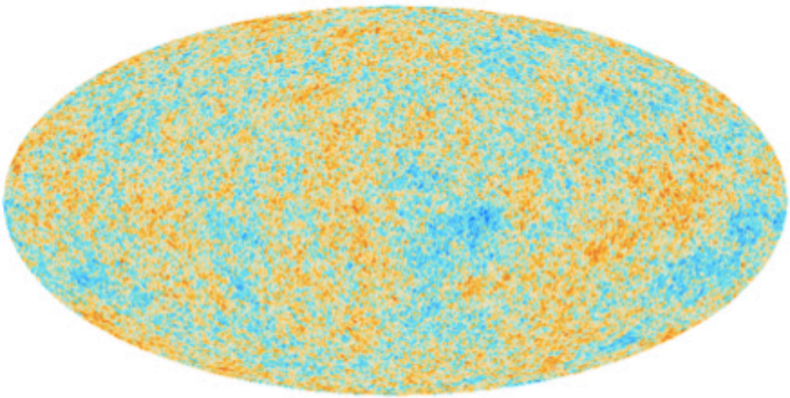


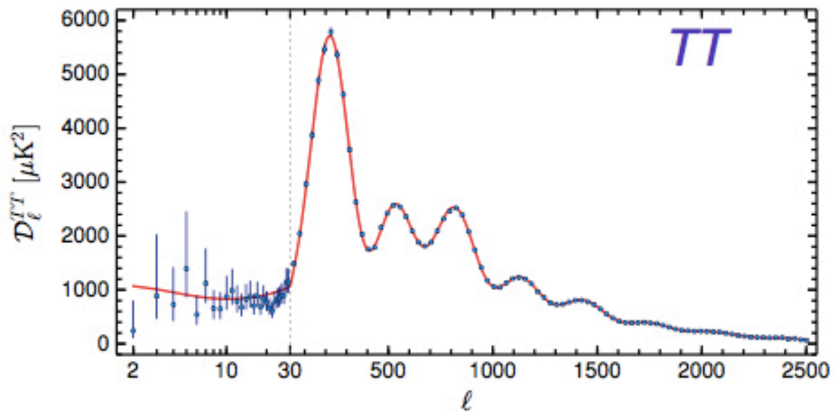
Last scattering surface

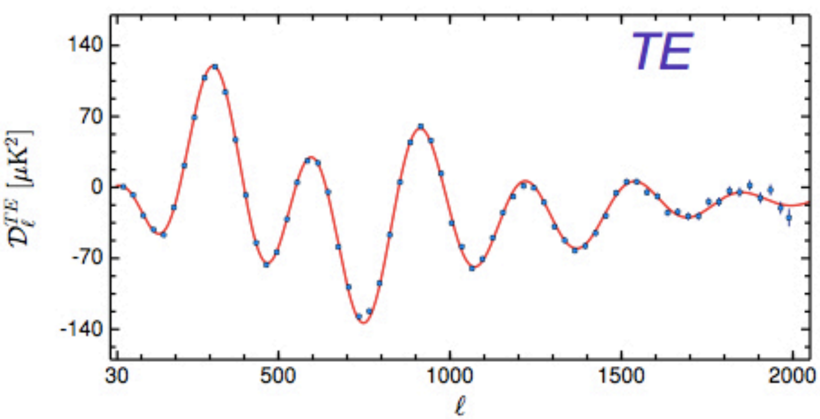

z=0
θ
14 000 Mpc
z~1100

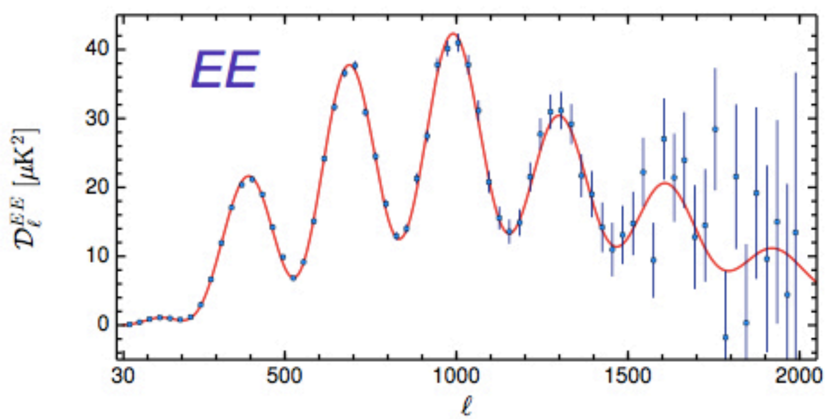
What can we learn from Planck 2015

Temperature anisotropies, T & E power spectra:





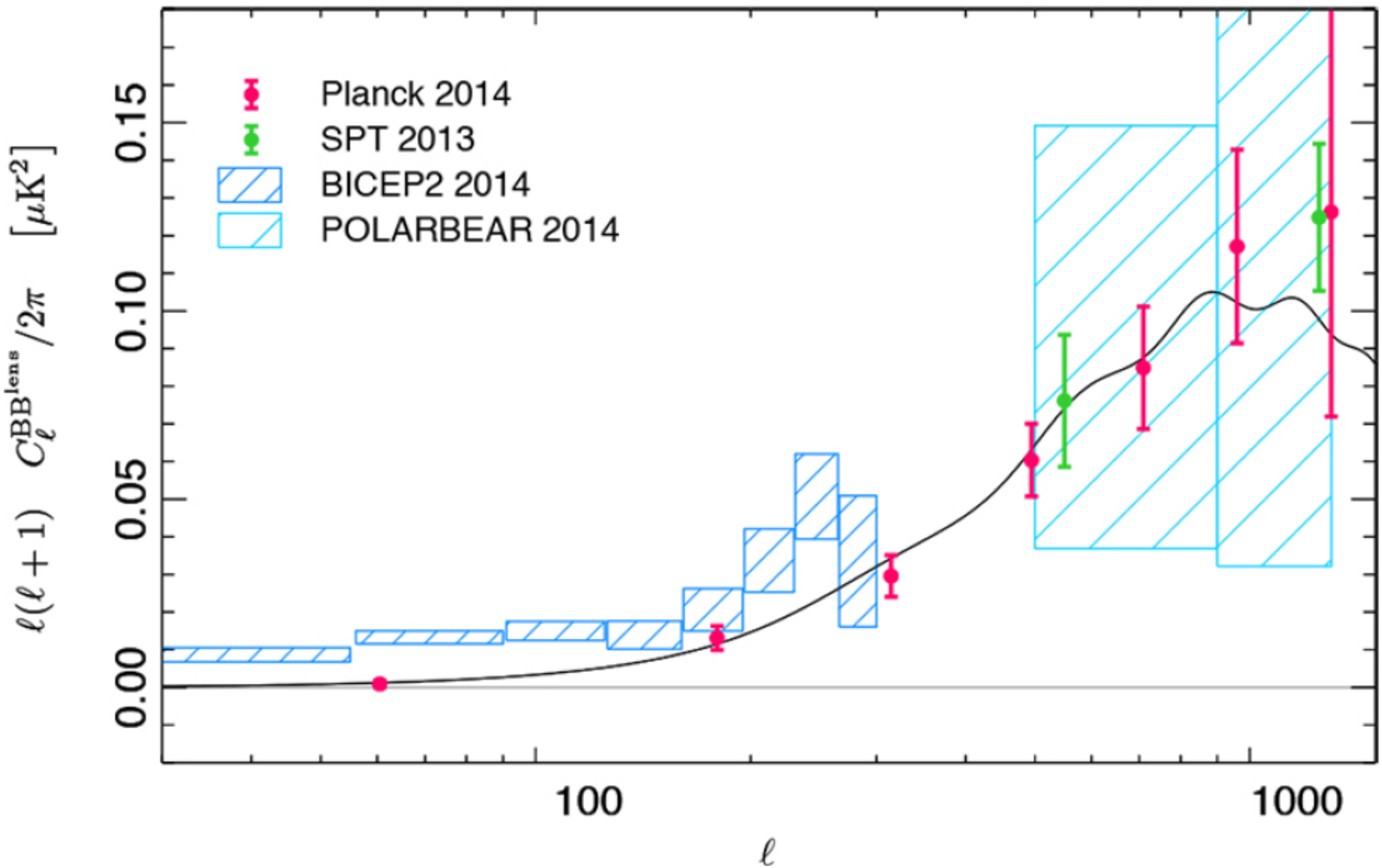




1502.01589

What can we learn from Planck 2015

Lensing induced B mode power spectrum:

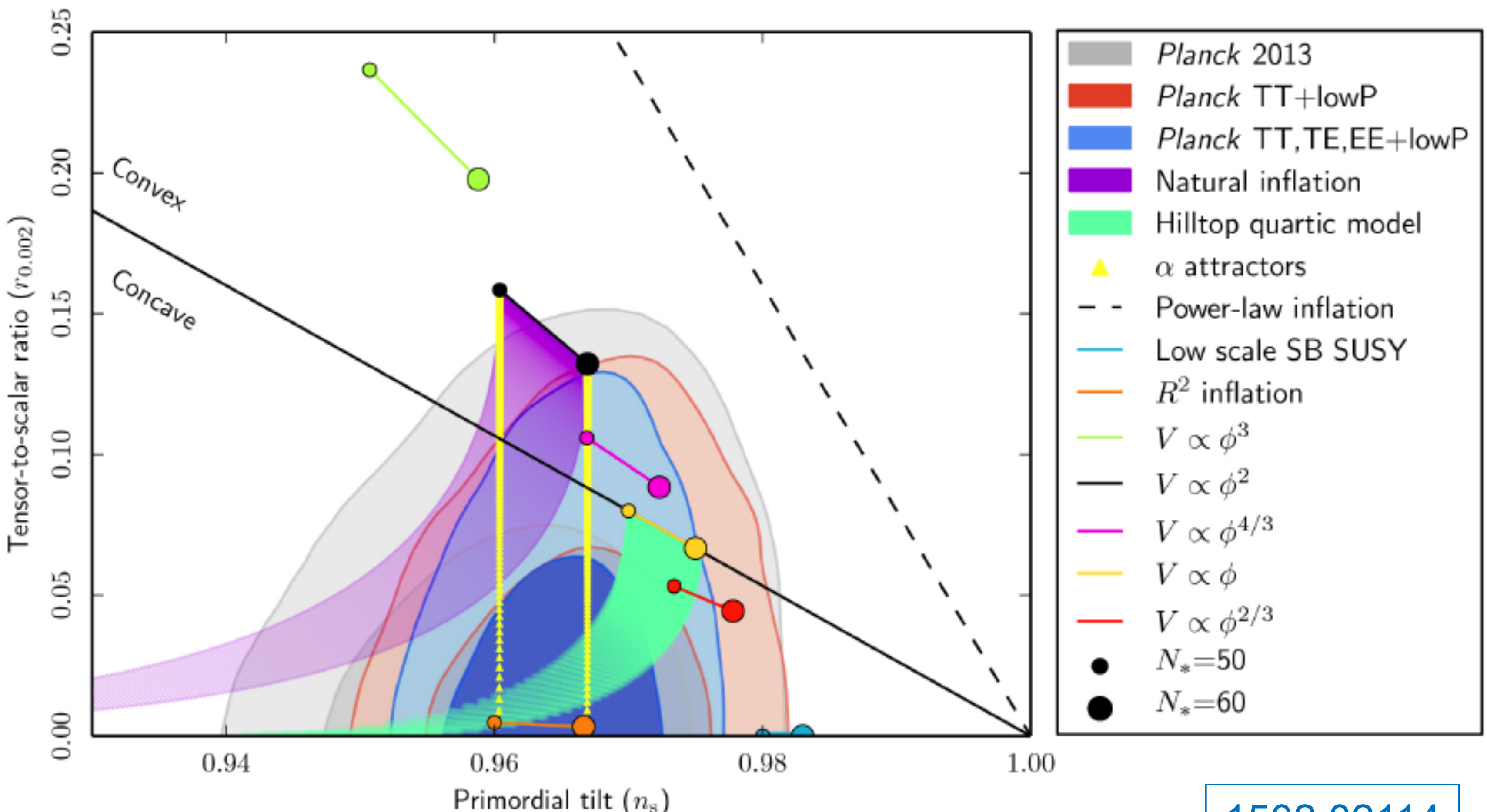


Concordance model: inflationary LCDM

$$\{H_0, \Omega_b, \Omega_c, A_s, n_s, \tau\}$$

- 7 peaks in 2013, 19 peaks in 2015;
- LCDM is perfect in explaining three CMB maps from l =30 until l=2000;
- A nearly scale-invariant, adiabatic, Gaussian power spectrum of primordial fluctuations as predicted by inflation seems highly consistent with data.

Planck 2015 constraints on inflation models



1502.02114

CMB Blackbody background

- CMB is a (nearly) perfect blackbody characterized by a phase space distribution function

$$f = \frac{1}{e^{E/T} - 1}$$

  where the temperature $T(x, \hat{n}, t)$ is observed at our position $x=0$ and time t_0 to be nearly isotropic with a mean temperature of 2.725K

- Our observable is the temperature anisotropy

$$\Theta(\hat{\mathbf{n}}) \equiv \frac{T(0, \hat{\mathbf{n}}, t_0) - \bar{T}}{\bar{T}}$$

- Given that physical processes essentially put a band limit on this function it is useful to decompose it into a complete set of harmonic coefficients

PGWs induce temperature fluctuations & polarization in CMB

- Consider that the universe is filled with CMB photons that do not scatter. Thus, the photon energies are affected only by the metric.

- As a toy model, consider a single monochromatic plane-wave GW, which appears as a tensor perturbation to the FRW metric:

$$ds^2 = a^2(\eta)\left[d\eta^2 - dx^2(1+h_+) + dy^2(1-h_+) + dz^2\right]$$

$$h_+(x,\eta) \simeq h(\eta)e^{ik\eta}e^{-ikz}$$

  which describes a linearly-polarized plane wave along the z axis.

- Photons that propagate freely through this spacetime experience a frequency shift dν in an expansion interval determined by the geodesic equation, which takes the form:

$$\frac{1}{\nu}\frac{d\nu}{d\eta} = -\frac{1}{2}(1-\mu^2)\cos 2\phi e^{-ikz}\frac{d}{d\eta}(he^{ik\eta})$$

  where μ is the cosine of the angle that photon trajectory makes with z axis, ϕ is the azimuthal angle of the photon's trajectory.

PGWs induce temperature fluctuations & polarization in CMB

- The above effect is polarization-independent, but polarization is induced by Thomson scattering of this anisotropic radiation field. To account for the polarization, we must follow the time evolution of four distribution functions:

$$f_s(\boldsymbol{x}, \boldsymbol{q}; \eta)$$

q is photon momentum; s=(I,Q,U,V) are four Stokes parameters

- At unperturbed background:

$$\bar{f}_I(\boldsymbol{q}, \boldsymbol{x}; \eta) = \left[e^{h\nu/k_B T(\eta)} - 1\right]^{-1} \qquad \bar{f}_Q = \bar{f}_U = \bar{f}_V = 0$$

- Then we introduce the perturbations

$$\Delta_s e^{i\boldsymbol{k}\cdot\boldsymbol{x}} = 4\delta f_s/(\partial \bar{f}/\partial \ln T)$$

$$\Delta_I = \tilde{\Delta}_I (1-\mu)^2 \cos 2\phi, \qquad \Delta_Q = \tilde{\Delta}_Q (1+\mu)^2 \cos 2\phi, \qquad \Delta_U = \tilde{\Delta}_U 2\mu \sin 2\phi,$$

which are variables as functions only of μ and time.

Harmonic Analysis for CMB Polarizations

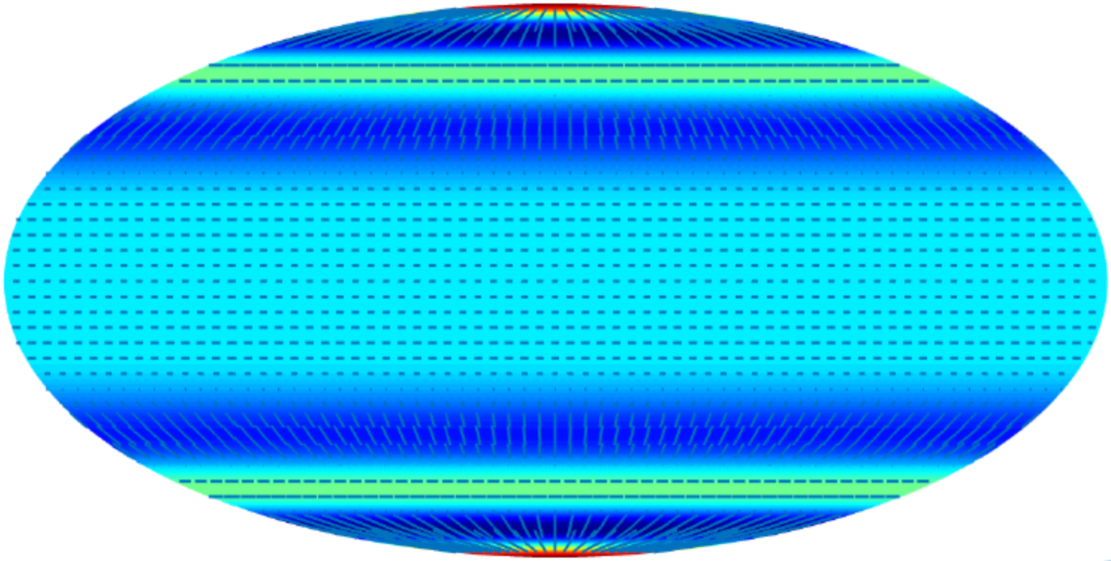
- Comments:
  - Thomson scattering induces no circular polarization, so $\Delta_V = 0$ at all times.
  - For GWs induced perturbations, there is: $\tilde{\Delta}_Q = -\tilde{\Delta}_U$ since the orientation of the photon polarization is fixed by the direction of the photon w.r.t. the GW polarization.
  - Consequently, there are only two Boltzmann equations:
    $$\dot{\tilde{\Delta}}_I + ik\mu\tilde{\Delta}_T = -\dot{h} - \dot{\kappa}\left[\tilde{\Delta}T - \Psi\right], \qquad \dot{\tilde{\Delta}}_Q + ik\mu\tilde{\Delta}_Q = -\dot{\kappa}\left[\Delta_P + \Psi\right]$$
    where the dot w.r.t. conformal time and Legendre moments:
    $$\tilde{\Delta}_{I\ell}(\eta) = (1/2)\int_{-1}^{1} d\mu\, \tilde{\Delta}_I(\mu;\eta)P_\ell(\mu)$$
  - The r.h.s. account for Thomson scattering.

astro-ph/9506072

Harmonic Analysis for CMB Polarizations

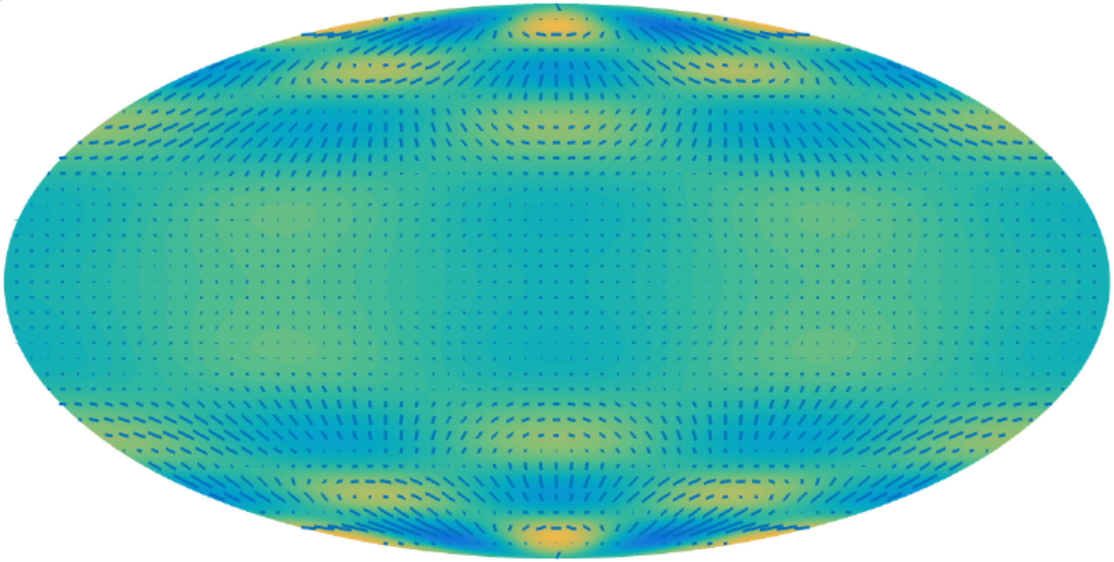
1510.06042

CMB temperature-polarization pattern induced by one Fourier mode of scalar perturbation (top) and one GW mode propagating in z direction (bottom):



Scalar mode

Tensor mode



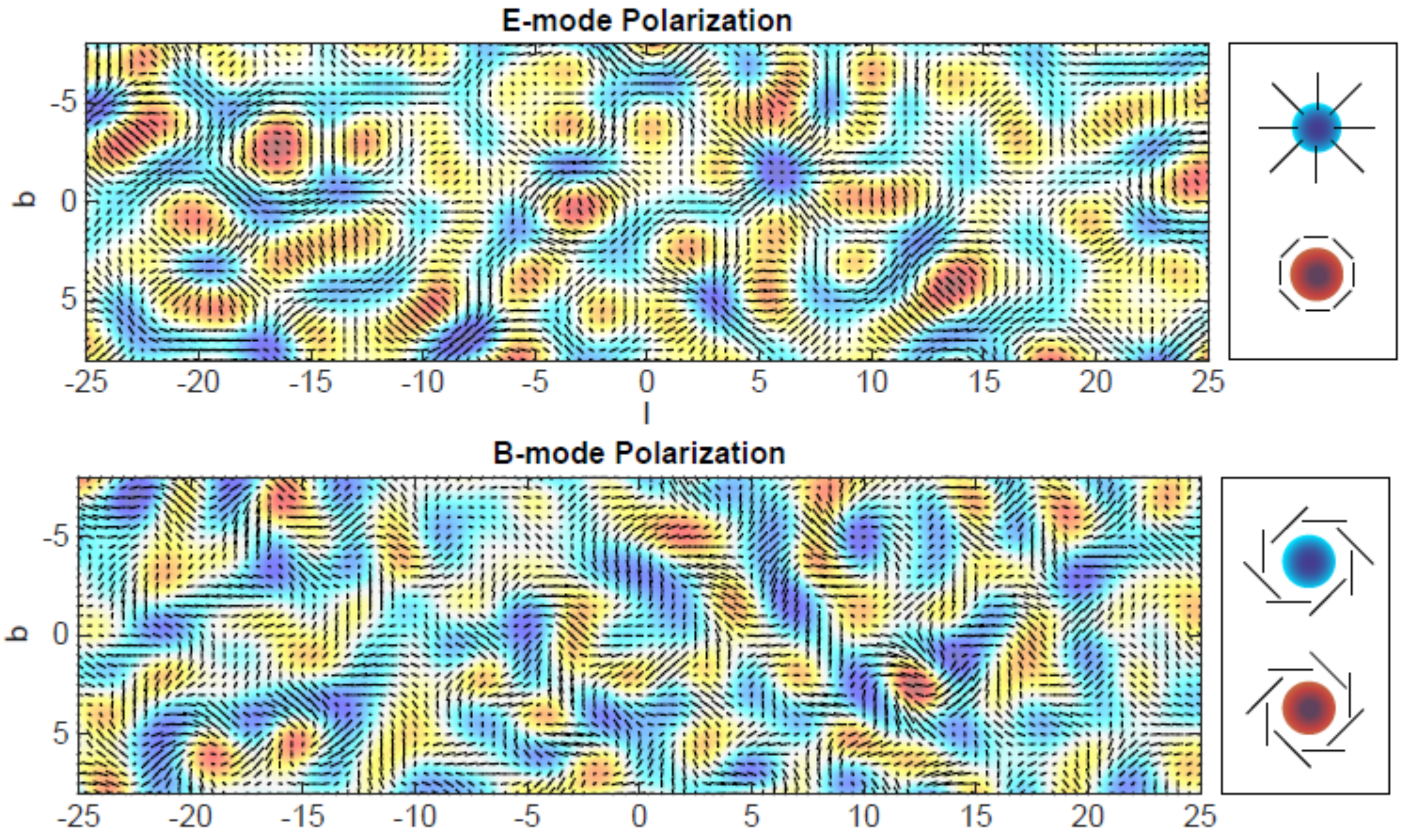
There are variations in U, the component of the polarization π/4 with respect to constant-longitude lines.

This is a signature of B mode induced by GW!

Harmonic Analysis for CMB Polarizations

- Note that the polarization is spin-2 field:

$$\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$



E and B modes from PGWs

- PGWs of k wave-number can induce the polarization tensor:

$$\mathcal{P}^{ab}_{k,+}(\theta,\phi) = \frac{T_0}{4\sqrt{2}} \sum_\ell (2\ell+1) P_\ell(\cos\theta) \tilde{\Delta}_{Q\ell} \begin{pmatrix} (1+\cos^2\theta)\cos 2\phi & 2\cot\theta \sin 2\phi \\ 2\cot\theta\sin 2\phi & -(1+\cos^2\theta)\csc^2\theta\cos 2\phi \end{pmatrix}$$

- It yields the E & B coefficients:

$$a^{\mathrm{E}\,k,+}_{\ell m} = \frac{\sqrt{\pi(2\ell+1)}}{4(\delta_{m,2}+\delta_{m,-2})^{-1}} \left[\frac{(\ell+2)(\ell+1)\tilde{\Delta}_{Q,\ell-2}}{(2\ell-1)(2\ell+1)} + \frac{6\ell(\ell+1)\tilde{\Delta}_{Q\ell}}{(2\ell+3)(2\ell-1)} + \frac{\ell(\ell-1)\tilde{\Delta}_{Q,\ell+2}}{(2\ell+3)(2\ell+1)} \right]$$

$$a^{\mathrm{B}\,k,+}_{\ell m} = \frac{-i}{2\sqrt{2}} \sqrt{\frac{2\pi}{(2\ell+1)}} (\delta_{m,2} - \delta_{m,-2}) \left[(\ell+2)\tilde{\Delta}_{Q,\ell-1} + (\ell-1)\tilde{\Delta}_{Q,\ell+1} \right]$$

- The angular power of B mode for fixing k takes:

$$C^{\mathrm{BB},\,k,+}_\ell = \frac{1}{2l+1} \sum_m |a^B_{\ell m}|^2 = \frac{\pi}{2} \left[\frac{\ell+2}{2\ell+1} \tilde{\Delta}_{Q,\ell-1} + \frac{\ell-1}{2\ell+1} \tilde{\Delta}_{Q,\ell+1} \right]^2$$

- Integrating out k, one gets the BB angular power spectrum

$$\boxed{C^{\mathrm{BB}}_\ell = \frac{1}{2\pi} \int k^2\, dk \left[\frac{\ell+2}{2\ell+1} \tilde{\Delta}_{Q,\ell-1}(k) + \frac{\ell-1}{2\ell+1} \tilde{\Delta}_{Q,\ell-1}(k) \right]^2}$$

E and B modes from PGWs

- Comments:
  - Similar process applies for C_l^{EE}
  - The E-B cross correlation vanishes for the standard model
  - Taking the same analysis, it is easy to see that scalar perturbation only produces T and E, simply due to the fact that density perturbations do not produce a curl at linear level
  - But, B modes may still arise from density perturbations at nonlinear order
  - Question: How large are these foreground contaminations?

Lensing induced B modes

- The most relevant nonlinear effect is weak gravitational lensing induced by (scalar type) density perturbations between us and the CMB surface of last scatter.

astro-ph/9803150

- The Stokes parameters displace along a given direction:

$$\begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm obs.} (\theta) = \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm ls} (\theta + \delta\theta) \simeq \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm ls} (\theta) + \delta\theta \cdot \nabla \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\rm ls} (\theta)$$

where $\delta\theta = \nabla\Phi$ is the lensing deflection along gravitational potential.

- If no PGWs, there is only E mode at LSS with:

$$\tilde{Q}(\ell) = 2\tilde{E}(\ell)\cos 2\varphi_\ell \qquad U(\ell) = -2E(\ell)\sin 2\varphi_\ell$$

Lensing induced B modes

- Gravitational deflection leads to

$$B(\boldsymbol{\ell}) = \frac{1}{2}[\sin 2\varphi_{\ell}\, \delta Q(\boldsymbol{\ell}) - \cos 2\varphi_{\ell}\, \delta U(\boldsymbol{\ell})] = \int \frac{d^2\boldsymbol{l}_1}{(2\pi)^2} [\boldsymbol{\ell}_1 \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}_1)] E(\boldsymbol{\ell}_1) \Phi(\boldsymbol{\ell} - \boldsymbol{\ell}_1) \sin 2\varphi_{\boldsymbol{\ell}_1}$$

- The angular power spectrum of lensing B modes takes

$$\boxed{C_{\ell}^{\mathrm{BB}} = \int \frac{d^2\boldsymbol{l}_1}{(2\pi)^2} [\boldsymbol{\ell}_1 \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}_1)]^2 \sin^2 2\varphi_{\boldsymbol{\ell}_1} C_{|\boldsymbol{\ell} - \boldsymbol{\ell}_1|}^{\Phi\Phi} C_{\ell_1}^{\mathrm{EE}}}$$

- It was for the first time detected by the SPT in 2013.

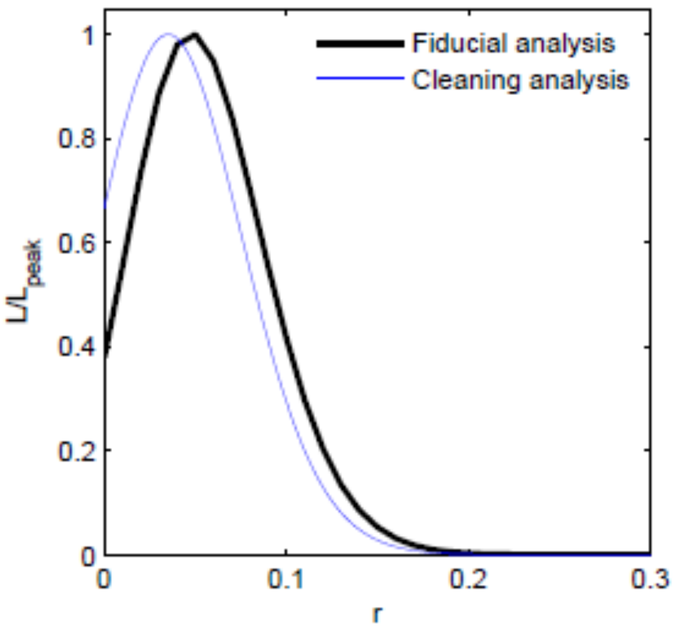
1307.5830

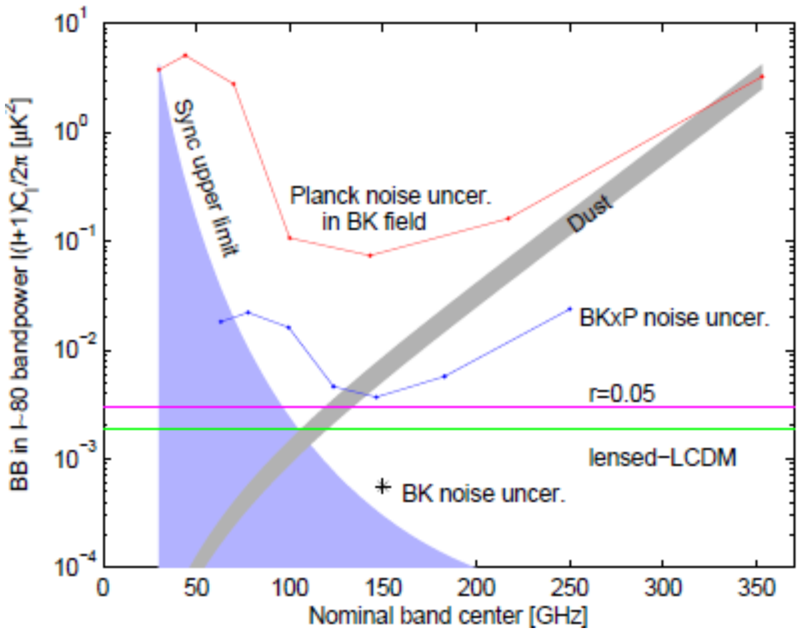
Foreground contributions to B modes

- Galactic foregrounds:
  - Synchrotron: Galactic synchrotron emission is dominant at frequencies below 100 GHz, and both WMAP and Planck have observed its polarization signature at frequencies from 30 to 90 GHz;
  - Dust: Above 100 GHz, thermal emission from asymmetric dust in the interstellar medium, which align themselves with the Galactic magnetic field, induces a strong polarization signal;
  - …
- One must use techniques of de-lensing and non-Gaussian diagnosis to eliminate foreground contaminations to extract signals of PGWs.

So far where we are ...

1502.00612





- No signal of PGWs: r < 0.07 at 2σ under a joint analysis of data from BICEP/Keck Array & Planck 2015.

- The project of Ali Cosmic microwave Polarization Telescope (AliCPT) is on the way!

Summary

❖ Part I: Theory of primordial gravitational waves

- The era of GW astronomy has been initiated
- PGWs are powerful to probe the very beginning of the universe
- Both inflation and matter bounce cosmologies are able to produce nearly scale-invariant power spectrum of PGWs
- These two cosmological paradigms are distinguishable on the test of consistency relations

Summary

- Part II: From PGWs to the CMB

  - CMB is a perfect black body, yet remains tiny temperature anisotropies

  - PGWs can affect CMB photons on their frequency and polarizations

  - Primordial B mode is the unique signal from PGWs

  - Late time evolutions can induce B modes nonlinearly, such as CMB lensing and other foreground contaminations

Outlook

- The next generation of CMB experiments are ongoing
- AliCPT will be the first ground-based CMB experiment in the north sphere
- The application of more detectors may provide more stringent constraints on models of very early universe
- New observational window implies new chance as well as new challenges

谢谢！